Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\frac{\sin 3x - \sin x}{\cos x - \cos 3x}$ is equal to **[1]**

a) cot 2x

b) - cot 2x

c) - tan 2x

d) tan 2x

2. Let f(x) = x^3 . Then, dom (f) and range (f) are respectively **[1]**

a) R and R^+

b) R^+ and R

c) R^+ and R^+

d) R and R

3. A batsman scores runs in 10 innings as 38, 70, 48, 34, 42, 55, 63, 46, 54 and 44. The mean deviation about mean is **[1]**

a) 7.6

b) 6.4

c) 8.6

d) 10.6

4. If y = $\sqrt{x+\sqrt{x+\sqrt{x+\ldots+to\infty}}}\infty$ then $\frac{dy}{dx}$ = **[1]**

a) $\frac{1}{2y+1}$

b) $\frac{1}{2y-1}$

c) $\frac{x}{y+1}$

d) $\sqrt{\frac{x}{y+1}}$

5. The two lines ax + by = c and a′x + b′y = c′ are perpendicular if **[1]**

a) ab′ = ba′

b) aa′ + bb′ = 0

c) ab + a′b′ = 0

d) ab′ + ba′ = 0

6. A plane is parallel to yz-plane so it is perpendicular to: **[1]**

a) y-axis

b) none of these

c) z-axis

d) x-axis

7. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then **[1]**

a) Re(z)=0

b) Re(z) > 0, I_mz) >

c) Re(z) > 0, I_{rs}(z) < 0

d) I_m(z)=0

8. 0! is always taken as **[1]**

a) 1

b) 2

c) ∞

d) 0

9. $\lim\limits_{x \to \infty} \frac{\sin x}{x} =$ **[1]**

a) 2

b) 1

c) ∞

d) 0

10. tan 15° = ? **[1]**

a) $\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)}$

b) $\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}$

c) $\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$

d) $\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}$

11. The set of all prime numbers is **[1]**

a) an infinite set

b) a singleton set

c) a multi set

d) a finite set

12. The integral part of $(\sqrt{2}+1)^6$ is **[1]**

a) 98

b) 96

c) 99

d) 100

13. $\sum_{r=0}^{n} 4^r . {}^nC_r$ is equal to **[1]**

a) 6^n

b) 5^{-n}

c) 4^n

d) 5^n

14. Solve the system of inequalities: $\frac{x+7}{x-8} > 2, \frac{2x+1}{7x-1} > 5$ **[1]**

a) (4, 8)

b) (3, 6)

c) no solution

d) (2, 5)

15. If A = {1, 2, 3, 4, 5, 6} then the number of proper subsets is **[1]**

a) 63

b) 36

c) 64

d) 25

16. cos 15° = ? **[1]**

a) $\frac{(\sqrt{3}+1)}{\sqrt{2}}$

b) $\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

c) $\frac{(\sqrt{3}-1)}{2\sqrt{2}}$

d) $\frac{(\sqrt{3}-1)}{\sqrt{2}}$

17. If $y = \frac{\sin x+\cos x}{\sin x-\cos x}$, then $\frac{dy}{dx}$ at x = 0 is equal to **[1]**

a) 0

b) -2

c) $\frac{1}{2}$

d) Does not exist

18. The total number of 9 digit numbers which have all different digits is **[1]**

a) 9!

b) $10 \times 10!$

c) 10!

d) $9 \times 9!$

19. **Assertion (A):** Let A = {a, b} and B = {a, b, c}. Then, A $\not\subset$ B. **[1]**

Reason (R): If A $\subset$ B, then A $\cup$ B = B.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** A sequence is said to definite if it has finite no of terms. **[1]**

Reason (R): The sequence whose n$^{\text{th}}$ term if $\frac{2^n}{n}$ if 2, 2, $\frac{8}{3}$, 4...

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. If A and B are any two non-empty sets, then prove that $A \times B = B \times A \Leftrightarrow A = B$ **[2]**

OR

Find the domain of $f(x) = \frac{1}{x+2}$.

22. Evaluate: $\lim\limits_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta}$. **[2]**

23. Two dice are thrown simultaneously. Find the probability of getting a total of at least 10. **[2]**

OR

If A and B are two events associated with a random experiment such that P(A) = 0.25, P(B) = 0.4 and P(A or B) = 0.5, find the values of

i. P(A and B)

ii. P(A and $\bar{B}$)

24. Is A = {x : x $\in$ N, 1 < x $\leq$ 2} null set? **[2]**

25. Find the coordinates of the point which divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7. **[2]**

Section C

26. Prove that : ${}^{2n}C_n = \frac{2^n[1\cdot3\cdot5.....(2n-1)]}{n!}$. **[3]**

27. Find the point in yz-plane which is equidistant from the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2). **[3]**

28. Using binomial theorem, expand: $(\sqrt{x} + \sqrt{y})^8$ **[3]**

OR

In the expansion of $(x + a)^n$, sums of odd and even terms are P and Q respectively, prove that

i. $2(P^2 + Q^2) = (x + a)^{2n} + (x - a)^{2n}$

ii. $P^2 - Q^2 = (x^2 - a^2)^n$

29. If f(x) = mx + c and f(0) = f'(0) = 1. What is value of f(2)? **[3]**

OR

Differentiate the function: 3^{x-5}

30. The sum of three numbers in G.P. is 14. If the first two terms are each increased by 1 and the third term decreased by 1, the resulting numbers are in A.P. Find the numbers. **[3]**

OR

The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

31. Out of 25 members in a family, 12 like to take tea, 15 like to take coffee and 7 like to take coffee and tea both. How many like **[3]**

i. at least one of the two drinks
ii. only tea but not coffee
iii. only coffee but not tea
iv. neither tea nor coffee

Section D

32. Find the mean and standard deviation for the following data: **[5]**

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	2	4	6	5	5	5	2	8	5

33. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse. **[5]**

$\frac{x^2}{100}+\frac{y^2}{400}=1$

OR

Find the lengths major and minor axes, coordinates of the vertices, coordinates of the foci, eccentricity, and length of the latus rectum of the ellipse $9x^2 + y^2 = 36$.

34. Solve the following system of linear inequalities. **[5]**

$2(2x + 3) - 10 < 6(x - 2)$

and $\frac{2x-3}{4}+6 \geq 2+\frac{4x}{3}$

35. If $\sin x = \frac{\sqrt{5}}{3}$ and x lies in the 2nd quadrant, find the values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$. **[5]**

OR

Prove that: $\cos 36^o \cos 42^o \cos 60^o \cos 78^o = \frac{1}{16}$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Representation of a Relation

A relation can be represented algebraically by roster form or by set-builder form and visually it can be represented by an arrow diagram which are given below

i. **Roster form** In this form, we represent the relation by the set of all ordered pairs belongs to R.
ii. **Set-builder form** In this form, we represent the relation R from set A to set B as R = {(a, b): a ∈ A, b ∈ B and the rule which relate the elements of A and B}.
iii. **Arrow diagram** To represent a relation by an arrow diagram, we draw arrows from first element to second element of all ordered pairs belonging to relation R.

Questions:

i. If n(A) = 3 and B = {2, 3, 4, 6, 7, 8} then find the number of relations from A to B. (1)
ii. If A = {a, b} and B = {2, 3}, then find the number of relations from A to B. (1)

iii. If A = {a, b} and B = {2, 3}, write the relation in set-builder form. (2)

OR

Express of R = {(a, b): 2a + b = 5; a, b ∈ W} as the set of ordered pairs (in roster form). (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Two students Ankit and Vinod appeared in an examination. The probability that Ankit will qualify the examination is 0.05 and that Vinod will qualify is 0.10. The probability that both will qualify is 0.02.

i. Find the probability that atleast one of them will qualify the exam. (1)
ii. Find the probability that atleast one of them will not qualify the exam. (1)
iii. Find the probability that both Ankit and Vinod will not qualify the exam. (2)

OR

Find the probability that only one of them will qualify the exam. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The conjugate of a complex number z, is the complex number, obtained by changing the sign of imaginary part of z. It is denoted by $\bar{z}$.

The modulus (or absolute value) of a complex number, z = a + ib is defined as the non-negative real number $\sqrt{a^2 + b^2}$. It is denoted by |z|. i.e.

$|z| = \sqrt{a^2 + b^2}$

Multiplicative inverse of z is $\frac{\bar{z}}{|z|^2}$. It is also called reciprocal of z.

$z\bar{z} = |z|^2$

i. If f(z) = $\frac{7-z}{1-z^2}$, where z = 1 + 2i, then find |f(z)|. (1)
ii. Find the value of(z + 3)($\bar{z}$ + 3). (1)
iii. If (x - iy) (3 + 5i) is the conjugate of -6 - 24i, then find the value of x + y. (2)

OR

If z = 3 + 4i, then find $\bar{z}$. (2)

Solution

Section A

1. **(a)** cot 2x

Explanation: Using sin C - sin D = 2 cos $\frac{(C+D)}{2}$ sin $\frac{(C-D)}{2}$

and cos C - cos D = $-2\sin\frac{(C+D)}{2}\ \sin\frac{(C-D)}{2}$, we get

$\frac{\sin 3x-\sin x}{\cos x-\cos 3x}=\frac{2\cos\left(\frac{4x}{2}\right)\sin\left(\frac{2x}{2}\right)}{2\sin\left(\frac{4x}{2}\right)\sin\left(\frac{2x}{2}\right)}=\frac{\cos 2x\sin x}{\sin 2x\sin x}$ = cot 2x.

2.

(d) R and R

Explanation: f(x) = x^3

f(x) can assume any value, so domain of f(x) is R

The Range of the function can be positive or negative Real numbers, as the cube of any number depends on the sign of the number, So Range of f(x) is R

3.

(c) 8.6

Explanation: First we arrange score in the ascending order

Then scores are : 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

As there are 10 items in this data,

So, median will be the mean of fifth and sixth term

∴ Median = $\frac{46+48}{2}$ = 47

Now, deviation from median for each value

d_i = 13, 9, 5, 3, 1, 1, 7, 8, 16, 23

∴ Required Mean deviation = $\frac{\sum d_i}{10}=\frac{86}{10}$ = 8.6

4.

(b) $\frac{1}{2y-1}$

Explanation: y = $\sqrt{(x+y)}$

$y^2 = x + y$

2yy' = 1 + y'

5.

(b) aa′ + bb′ = 0

Explanation: We know that Slope of the line ax + by = c is $\frac{-a}{b}$, and the slope of the line a′x + b′y = c′ is $\frac{-a'}{b'}$ The lines are perpendicular if $\tan\theta=\frac{3}{5-x}$ (1)

$\frac{-a}{b}\frac{-a'}{b'}=-1$ or $aa'+bb'=0$

6.

(d) x-axis

Explanation: Any plane parallel to yz-plane is perpendicular to x-axis.

7.

(d) $I_m(z)=0$

Explanation: From given ,

$z=2^5C_0\frac{\sqrt{3}}{2}^2+^5C_2\frac{\sqrt{3}}{2}^3\frac{i^2}{2}+^5C_4\frac{\sqrt{3}}{2}\frac{i}{2}$

Since t^2 = -1 and t^4 = 1, will not contain any i and hence $I_m(z)=0$

8. **(a)** 1

Explanation: We have $^nP_r=\frac{n!}{(n-r)!}$ .....(i)

Number of ways you can arrange n thing in n available spaces = n!

$\Rightarrow {}^nP_n = n!$ ....(ii)

But from (i) we get ${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$ .....(iii)

Now from (ii) and (iii) we get $\frac{n!}{0!}$ = n! $\Rightarrow$ 0! = 1

9.

(d) 0

Explanation: $\lim\limits_{x\to\infty} \frac{\sin x}{x}$

Let $x = \frac{1}{y}$

$x \to \infty$

$\therefore y \to 0$

$= \lim\limits_{y\to 0} \frac{\sin\frac{1}{y}}{\frac{1}{y}}$

$= \lim\limits_{y\to 0} y \sin\frac{1}{y}$

$= \lim\limits_{y\to 0} y \times \lim\limits_{y\to 0} \sin\frac{1}{y}$

$= 0 \times \lim\limits_{y\to 0} \sin\frac{1}{y}$

= 0

10.

(c) $\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$

Explanation: tan 15° = tan(45° - 30°) $= \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ} = \frac{\left(1-\frac{1}{\sqrt{3}}\right)}{\left(1+\frac{1}{\sqrt{3}}\right)} = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$

11. **(a)** an infinite set

Explanation: Set A = {2, 3, 5, 7,...} so it is infinite.

12.

(c) 99

Explanation: We have $(1 + x)^n = 1 +^n C_1(x) +^n C_2(x)^2 + ...+ (x)^n$

Hence $(\sqrt{2}+1)^6 = 1 + {}^6C_1(\sqrt{2}) +^6 C_2(\sqrt{2})^2 +^6 C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^4 +^6 C_5(\sqrt{2})^5 + (\sqrt{2})^6$

$\Rightarrow (\sqrt{2}+1)^6 = 1 + 6(\sqrt{2}) + 15 \times 2 + 20 \times 2(\sqrt{2}) + 15 \times 4 + 6 \times 4(\sqrt{2}) + 8$

$= 99 + 70\sqrt{2}$

Hence integral part of $(\sqrt{2}+1)^6 = 99$

13.

(d) 5^n

Explanation: $\sum_{r=0}^{n} 4^r.^n C_r = 4^0 .^n C_0 + 4^1 .^n C_1 + 4^2 .^n C_2 + ... + 4^n .^n C_n$

$= 1 + 4.^n C_1 + 4^2.^n C_2 + ... + 4^n .^n C_n$

$= (1 + 4)^n = 5^n$

14.

(c) no solution

Explanation: $\frac{x+7}{x-8} > 2$

$\Rightarrow \frac{x+7}{x-8} - 2 > 0$

$\Rightarrow \frac{x+7-2(x-8)}{x-8} > 0$

$\Rightarrow \frac{x+7-2x+16}{x-8} > 0$

$\Rightarrow \frac{(23-x)}{x-8} > 0$ $[\because \frac{a}{b} > 0 \Rightarrow (a > 0 \text{ and } b > 0) \text{ or } (a < 0 \text{ and } b < 0)]$

$\Rightarrow$ (23 - x > 0 and x - 8 > 0) or (23 - x < 0 and x - 8 < 0)

$\Rightarrow$ (x < 23 and x > 8) or (x > 23 and x < 8)

$\Rightarrow$ 8 < x < 23 [Since x > 23 and x < 8 is not possible]

$\Rightarrow x \epsilon (8, 23)$

Now $\frac{2x+1}{7x-1} > 5$

$\Rightarrow \frac{2x+1}{7x-1}$ - 5 > 0

$\Rightarrow \frac{2x+1-5(7x-1)}{7x-1}$ > 0

$\Rightarrow \frac{2x+1-35x+5}{7x-1}$ > 0

$\Rightarrow \frac{(6-33x)}{7x-1}$ > 0 [$\because \frac{a}{b}$ > 0 $\Rightarrow$ (a > 0 and b > 0) or (a < 0 and b < 0)]

$\Rightarrow$ (6 - 33x > 0 and 7x - 1 > 0) or (6 - 33x < 0 and 7x - 1 < 0)

$\Rightarrow$ (x < $\frac{6}{33}$ and x > $\frac{1}{7}$) or (x > $\frac{2}{11}$ and x < $\frac{1}{7}$)

$\Rightarrow \frac{1}{7} < x < \frac{2}{11}$

$\Rightarrow x \in \left(\frac{1}{7}, \frac{2}{11}\right)$ [Since x > $\frac{2}{11}$ and x < $\frac{1}{7}$ is not possible]

Hence, the solution of the system $\frac{x+7}{x-8} > 2, \frac{2x+1}{7x-1} > 5$ will be $(8,23) \cap \left(\frac{1}{7}, \frac{2}{11}\right) = \phi$

15. **(a)** 63

Explanation: 63

The no. of proper subsets = 2^n - 1

Here n(A) = 6

In case of the proper subset, the set itself is excluded that's why the no. of the subset is 63. But if it is asked no. of improper or just no. of subset then you may write 64

So no. of proper subsets = 63

16.

(b) $\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

Explanation: cos 15° = cos (45° - 30°) = cos 45°cos 30° + sin 45° sin 30°

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) = \frac{(\sqrt{3}+1)}{2\sqrt{2}}$

17.

(b) -2

Explanation: Given, $y = \frac{\sin x+\cos x}{\sin x-\cos x}$

$\frac{dy}{dx} = \frac{(sinx-cosx)(cosx-sinx)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$

$= \frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$

$= \frac{-[\sin^2 x+\cos^2 x-2\sin x\cos x+\sin^2 x+\cos^2 x+2\sin x\cos x]}{(\sin x-\cos x)^2}$

$= \frac{-2}{(\sin x-\cos x)^2}$

$\therefore \left(\frac{dy}{dx}\right)_{\text{at } x=0} = \frac{-2}{(\sin 0-\cos 0)^2} = \frac{-2}{(-1)^2} = -2$

18.

(d) 9 × 9!

Explanation: We have to form 9 digit numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and we know that 0 can not be put on extremely left place. Therefore, first place from the left can be filled in 9 ways.

Now repetition is not allowed. Therefore, the remaining 8 places can be filled in 9!

∴ The required number of ways = 9 × 9!

19.

(d) A is false but R is true.

Explanation: Assertion A = {a, b}, B = {a, b, c}

Since, all the elements of A are in B. So,

A ⊂ B

Reason ∵ A ⊂ B

⇒ A ∪ B = B

Hence, Assertion is false and Reason is true.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion is true.

Reason

Let $t_n = \frac{2^n}{n}$

Putting n = 1, 2, 7, x

$t_1 = 2$, $t_2 = 2$, $t_3 = \frac{8}{3}$, $t_4 = x$

80 the sequence is 2, 2, $\frac{8}{3}$, 4

Reason is also correct but not the correct explanation for Assertion.

Section B

21. First, let A = B. Then, we have to prove that $A \times B = B \times A$

Now, A = B

$\Rightarrow \quad A \times B = A \times A$ and $B \times A = A \times A$

$\Rightarrow \quad A \times B = B \times A$

Conversely, let $A \times B = B \times A$. Then, we have to prove that A = B.

Let x be an arbitrary element of A. Then,

$x \in A$

$\Rightarrow \quad (x, b) \in A \times B$ for all b∈B.

$\Rightarrow \quad (x, b) \in B \times A$

$\Rightarrow \quad x \in B$

$\therefore \quad A \subseteq B$

Let y be an arbitrary element of A. Then,

$y \in B$

$\Rightarrow \quad (a, y) \in A \times B$ for all $a \in A$

$\Rightarrow \quad (a, y) \in B \times A$

$\Rightarrow \quad y \in A$

$\therefore \quad B \subseteq A$

Hence, A = B.

Hence, $A \times B = B \times A \Leftrightarrow A = B$

OR

Here f (x) $= \frac{1}{x+2}$

f(x) assume real values for all real values of x except for x + 2 = 0 i.e. x = - 2.

Thus domain of f (x) =R - {-2}.

22. We have:$\lim\limits_{\theta \to 0} \left[\frac{1-\cos(4\theta)}{1-\cos 6\theta}\right]$

$\lim\limits_{\theta \to 0} \left[\frac{2\sin^2 2\theta}{2\sin^2 3\theta}\right]$ $\{\because 1 - \cos A = 2\sin^2\left(\frac{A}{2}\right)\}$

$\lim\limits_{\theta \to 0} \left[\frac{\sin^2 2\theta}{(2\theta)^2} \times \frac{(2\theta)^2}{\frac{\sin^2 3\theta}{(3\theta)^2} \times (3\theta)^2}\right]$

$\lim\limits_{\theta \to 0} \left[\left(\frac{\sin 2\theta}{2\theta}\right)^2 \times \left(\frac{3\theta}{\sin 3\theta}\right)^2 \times \frac{4}{9}\right]$

$= \frac{4}{9}\left[\because \lim\limits_{X \to 0} \frac{\sin x}{x} = 1\right]$

23. We know that in a single throw of two dice, the total number of possible outcomes is (6 × 6) = 36.

Let S be the sample space of the event and is given by

n(S) = 36.

Let E_5 = event of getting a total of at least 10. Then,

E_5 = event of getting a total of 10 or 11 or 12 = ((4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}.

$\Rightarrow$ n(E_5) = 6

$\therefore$ P(E_3) = $\frac{n(E_5)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

OR

i. It is given that

: P(A) = 0.25, P(A or B) = 0.5 and P(B) = 0.4

To find : P(A and B)

Formula used : P(A or B) = P(A) + P(B) - P(A and B)

Substituting the value in the above formula we get, 0.5 = 0.25 + 0.4 - P(A and B)

0.5 = 0.65 - P(A and B)

P(A and B) = 0.65 - 0.5

P(A and B) = 0.15

ii. Given : P(A) = 0.25, P(A and B) = 0.15 (from part (i))

To find : P(A and $\bar{B}$)

Formula used : P(A and $\bar{B}$) = P(A) - P(A and B)

Substituting the value in the above formula we get,

P(A and $\bar{B}$) = 0.25 - 0.15

P(A and $\bar{B}$) = 0.10

P(A and $\bar{B}$) = 0.10

24. We know that,Natural numbers = 1, 2, 3, 4, 5, 6, 7,…

Natural number greater than 1 ($1 < x$) = 2, 3, 4, 5, ...

Natural number less than or equal to 2 ($x \leq 2$) = 2

$\Rightarrow$ one element in this set

$\therefore$ It is not a null set.

25. Let P(x, y) be the point that divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7

We know that: If $m_1 : m_2$ is the ratio in which the join of two points is divided by another point (x, y), then

$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

$y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

Here, $x_1 = -5$, $x_2 = 4$, $y_1 = 11$ and $y_2 = -7$

Substituting,we get

$x = \frac{2\times4+7\times-5}{2+7}$

$x = \frac{8-35}{9}$

$x = \frac{-27}{9}$

$\Rightarrow$ x = -3

$y = \frac{2\times-7+7\times11}{2+7}$

$y = \frac{-14+77}{9}$

$y = \frac{63}{9}$

$\Rightarrow$ y = 8

Thus, the coordinates of the point which divided the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7 is (-3, 8).

Section C

26. ${}^{2n}C_n = \frac{2^n[1\cdot35.....(2n-1)]}{n!}$

$= \frac{(2n)!}{n!n!}$

$= \frac{\frac{(2n)(2n-1)(2n-2)(2n-3).....}{4\cdot3\cdot2\cdot1}}{n!n!}$

$= \frac{[2^n....4\cdot2][(2n-1)....3\cdot1]}{n!n!}$

$= \frac{2^n[1\cdot2\cdot....n][1\cdot3\cdot5.....\cdot(2n-1)]}{n!n!}$

$= \frac{2^n\times n![1\cdot3\cdot5....\cdot(2n-1)]}{n!n!}$

$= \frac{2^n[1\cdot3\cdot5....\cdot(2n-1)]}{n!}$

27. The general point on yz plane is D(0, y, z).

Consider this point is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2).

$\therefore$ AD = BD

$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2} = \sqrt{(0-1)^2+(y+1)^2+(z-0)^2}$

Squaring both sides,

$(0 - 3)^2 + (y - 2)^2 + (z + 1)^2 = (0 - 1)^2 + (y + 1)^2 + (z - 0)^2$

$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 1 + y^2 + 2y + 1 + z^2$

-6y + 2z + 12 = 0 ….(1)

Also, AD = CD

$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2} = \sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$

Squaring both sides,

$(0 - 3)^2 + (y - 2)^2 + (z + 1)^2 = (0 - 2)^2 + (y - 1)^2 + (z - 2)^2$

$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 4 + y^2 - 2y + 1 + z^2 - 4z + 4$

-2y + 6z + 5 = 0 ….(2)

By solving equation (1) and (2) we get

$y = \frac{31}{16}$ $z = \frac{-3}{16}$

The point which is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2) is $(\frac{31}{16}, \frac{-3}{16})$.

28. We hand to find value of $(\sqrt{x} + \sqrt{y})^8$

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(\sqrt{x} + \sqrt{y})^8$

We can write $\sqrt{x}$ as $x^{\frac{1}{2}}$ and $\sqrt{y}$ as $y^{\frac{1}{2}}$

Now, we have to solve for $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^8$

$$= \left[8C_0\left(x^{\frac{2}{2}}\right)^{8-0}\right] + \left[8C_1\left(x^{\frac{1}{2}}\right)^{8-1}\left(y^{\frac{2}{2}}\right)^1\right] + \left[8C_2\left(x^{\frac{1}{2}}\right)^{8-2}\left(y^{\frac{1}{2}}\right)^2\right] + \left[{}^8C_3\left(x^{\frac{1}{2}}\right)^{8-3}\left(y^{\frac{1}{2}}\right)^3\right]$$

$$+ \left[{}^8C_4\left(x^{\frac{1}{2}}\right)^{8-4}\left(\frac{1}{y^2}\right)^4\right] + \left[{}^8C_5\left(x^{\frac{1}{2}}\right)^{8-5}\left(y^{\frac{2}{2}}\right)^5\right] + \left[{}^8C_6\left(x^{\frac{1}{2}}\right)^{8-6}\left(y^{\frac{1}{2}}\right)^6\right]$$

$$+ \left[8C_7\left(x^{\frac{1}{2}}\right)^{8-7}\left(y^{\frac{1}{2}}\right)^7\right] + \left[8C_8\left(y^{\frac{1}{2}}\right)^8\right]$$

$$= \left[\frac{8!}{0!(8-0)!}\left(x^{\frac{5}{2}}\right)\right] + \left[\frac{8!}{1!(8-1)!}\left(x^{\frac{2}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + \left[\frac{8!}{2!(8-2)!}\left(x^{\frac{6}{2}}\right)\left(y^{\frac{2}{2}}\right)\right]$$

$$+ \left[\frac{8!}{3!(8-3)!}\left(x^{\frac{5}{2}}\right)\left(y^{\frac{3}{2}}\right)\right] + \left[\frac{8!}{4!(8-4)!}\left(x^{\frac{4}{2}}\right)\left(y^{\frac{4}{2}}\right)\right]$$

$$+ \left[\frac{8!}{5!(8-5)!}\left(x^{\frac{2}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] + \left[\frac{8!}{6!(8-6)!}\left(x^{\frac{2}{2}}\right)\left(\frac{6}{y^2}\right)\right] + \left[\frac{8!}{7!(8-7)!}\left(x^{\frac{1}{2}}\right)\left(\frac{2}{y^2}\right)\right] + \left[\frac{8!}{8!(8-8)!}\left(y^{\frac{5}{2}}\right)\right]$$

$$= \left[1\left(x^4\right)\right] + \left[8\left(x^{\frac{7}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + \left[28\left(x^3\right)(y)\right] + \left[56\left(x^{\frac{5}{2}}\right)\left(\frac{2}{y^2}\right)\right]$$

$$+ \left[70\left(x^2\right)\left(y^2\right)\right] + \left[56\left(x^{\frac{3}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] + \left[28\left(x^2\right)\left(y^3\right)\right] + \left[8\left(x^{\frac{1}{2}}\right)\left(y^{\frac{2}{2}}\right)\right] + \left[1\left(y^4\right)\right]$$

OR

Here $(x + a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \ldots + {}^nC_na^n$

= P + Q . . . (i)

where $P = {}^nC_0x^n + {}^nC_3x^{n-3}a^3 + \ldots.$

$Q = {}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + \ldots.$

Also $(x - a)^n = {}^nC_0x^n - {}^nCx^{n-1}a + {}^nC_2x^{n-2}a^2 + \ldots + (-1)^{n\,n}C_na^n \ldots$ (ii)

= P - Q

(i) Squaring and adding (i) and (ii) we have

$(x + a)^{2n} + (x - a)^{2n} = (P + Q)^2 + (P - Q)^2$

$= P^2 + Q^2 + 2PQ + P^2 + Q^2 - 2PQ$

$= 2P^2 + 2Q^2 = 2(P^2 + Q^2)$

(ii) Multiplying(i) and (ii) we have

$(x + a)^n (x - a)^n = (P + Q)(P - Q)$

$(x^2 - a^2)^n = P^2 - Q^2$

29. We have,

f(x) = mx + c ....(i)

Differentiating with respect to x, we get

f'(x) = m.1 + 0

$\Rightarrow$ f'(x) = m ....(ii)

Put, x = 0 in (i) and (ii), we get

f(0) = c and f'(0) = m

⇒ 1 = c and 1 = m [∴ f(0) = f'(0) = 1]

Put the values of m and c in f(x) = mx + c, we get f(x) = x + 1.

∴ f(2) = 2 + 1 = 3. [Put x = 2 in f(x) = x +1]

OR

We have,

$\frac{d}{dx}x^n = nx^{n-1}$

Therefore,

$\frac{d}{dx}3x^{-5}$= 3(-5)x^{-5-1}

= -15x^{-6}

30. Let three number in G.P. are $\frac{a}{r}$, a, ar

Here,

$\frac{a}{r} \times a \times ar = 729$

$\Rightarrow \quad a^3 = 729$

⇒ a = 9

From the given conditions we can write ,

$\left(\frac{a}{r} \times a\right) + (a \times ar) + \left(\frac{a}{r} \times ar\right) = 819$

$\Rightarrow \quad \frac{81}{r} + 81r + 81 = 819$

$\Rightarrow \quad \frac{9}{r} + 9r + 9 = 91$

⇒ 9 + 9r^2 + 9r = 91r

⇒ 9r^2 - 82r + 9 = 0

⇒ 9r^2 - 81r - r + 9 = 0

⇒ 9r(r - 9) -1(r - 9) = 0

r = 9, $\frac{1}{9}$

So, required G.P. are

81, 9, 1, ...

or, 1, 9, 81, ...

OR

Let $\frac{a}{r}$, a, ar be first three terms of the given G.P.

$\frac{a}{r} + a + ar = \frac{39}{10}$ ...(i)

($\frac{a}{r}$)(a)(ar) = 1 ...(ii)

From (ii) we obtain a^3 = 1 ⇒ a = 1 (considering real roots only)

Substituting a = 1 in equation (i), we obtain

$\frac{1}{\mathrm{r}} + 1 + \mathrm{r} = \frac{39}{10}$

$\Rightarrow 1 + \mathrm{r} + \mathrm{r}^2 = \frac{39}{10}\mathrm{r}$

⇒ 10 + 10r + 10r^2 - 39r = 0

⇒ 10r^2 - 29r + 10 = 0

⇒ 10r^2 - 25r - 4r + 10 = 0

⇒ 5r(2r - 5)-2(2r - 5) = 0

⇒ (5r - 2)(2r - 5) = 0

$\Rightarrow \mathrm{r} = \frac{2}{5}$ or $\frac{5}{2}$

corresponding terms of the G.P

i. when r = $\frac{2}{5}$

$\Rightarrow \frac{5}{2}, 1, \frac{2}{5}$

ii. when r = $\frac{5}{2}$

$\Rightarrow \frac{2}{5}, 1, \frac{5}{2}$

31. Given that, n(T) = 12

n(C) = 15

n(T ∩ C) = 7

i. n(T ∪ C) = n(T) + n(C) - n(T ∩ C)

= 12 + 15 - 7

$n(T \cup C) = 20$

20 members like at least one of the two drinks.

ii. Only tea but not coffee

$= n(T) - n(T \cap C)$

$= 12 - 7$

$= 5$

iii. Only coffee but not tea

$= n(C) - n(T \cap C)$

$= 15 - 7$

$= 8$

iv. Neither tea nor coffee

$= n(U) - n(T \cup C)$

$= 25 - 20$

$= 5$

Section D

32. We make the table from the given data:

Class marks	Mid value (x_i)	$d_i = x_i - a = x_i - 45$	f_i	$f_i d_i$	d_i^2	$f_i d_i^2$
0-10	5	-40	3	-120	1600	4800
10-20	15	-30	2	-60	900	1800
20-30	25	-20	4	-80	400	1600
30-40	35	-10	6	-60	100	600
40-50	45	0	5	0	0	0
50-60	55	10	5	50	100	500
60-70	65	20	5	100	400	2000
70-80	75	30	2	60	900	1800
80-90	85	40	8	320	1600	12800
90-100	95	50	5	250	2500	12500
			$\sum f_i = 45$	$\sum f_i d_i = 460$		$\sum f_i d_i^2 = 38400$

Let a = 45.

$\therefore$ Mean $= a + \frac{\sum f_i d_i}{\sum f_i}$

$= 45 + \frac{460}{45}$

$= 45 + 10.22 = 55.22$

$\therefore$ Standard deviation $= \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$

$= \sqrt{\frac{38400}{45} - (10.22)^2}$

$= \sqrt{853.33 - 104.45}$

$= \sqrt{748.88}$

$= 27.36$

33. The equation of given ellipse is $\frac{x^2}{100} + \frac{y^2}{400} = 1$

Now $400 > 100 \Rightarrow a^2 = 400$ and $b^2 = 100$

So the equation of ellipse in standard form is $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$

$\therefore a^2 = 400 \Rightarrow a = 20$ and $b^2 = 100 \Rightarrow b = 10$

We know that $c = \sqrt{a^2 - b^2}$

$\therefore c = \sqrt{400 - 100} = \sqrt{300} = 10\sqrt{3}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ i.e. $(0, \pm 10\sqrt{3})$

Coordinates of vertices are $(0, \pm a)$ i.e. $(0, \pm 20)$

Length of major axis = 2 a $= 2 \times 20 = 40$
Length of minor axis = 2 b = $2 \times 10 = 20$
Eccentricity (e) $= \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$
Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times100}{20} = 10$

OR

Given that:

$9x^2 + y^2 = 36$.

After divide by 36 to both the sides, we get

$\frac{9}{36}x^2 + \frac{1}{36}y^2 = 1 \Rightarrow \frac{x^2}{4} + \frac{y^2}{36} = 1$ ... (i)

Now, the above equation is of the form,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ... (ii)

Comparing Eq. (i) and (ii), we get

$a^2 = 36$ and $b^2 = 4 \Rightarrow a = \sqrt{36}$ and $b = \sqrt{4} \Rightarrow$ and a = 6 and b = 2

i. Length of major axes
   ∴ Length of major axes = 2a = 2 × 6 = 12 units
ii. Length of major axes
   ∴Length of major axes = 2b = 2 ×2 =4 units
iii. Coordinates of the Vertices
   ∴ Coordinates of the Vertices = (0, a) and (0, -a) = (0, 6) and (0, -6)
iv. Coordinates of the foci

   As we know that, Coordinates of foci $= (0, \pm c)$ where $c^2 = a^2 - b^2$

   Now

   $c^2 = 36 - 4 \Rightarrow c^2 = 32 \Rightarrow c = \sqrt{32}$ ... (iii)

   ∴ Coordinates of foci $= (0, \pm\sqrt{32})$
v. Eccentricity

   As we know that, Eccentricity $= \frac{c}{a} \Rightarrow e = \frac{\sqrt{32}}{6}$ [from (iii)]
vi. Length of the Latus Rectum

   As we know that, Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times(2)^2}{6} = \frac{8}{6}$
   $= \frac{4}{3}$

34. The given system of linear inequalities is

2(2x + 3) -10 < 6(x - 2) ....(i)

and $\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3}$ ...(ii)

From inequality (i), we get

2(2x + 3) - 10 < 6(x - 2)

⇒ 4x + 6 - 10 < 6x - 12

⇒ 4x - 4 < 6x - 12

⇒ 4x - 4 + 4 < 6x - 12 + 4 [adding 4 on both sides]

⇒ 4x < 6x - 8

⇒ 4x - 6x < 6x - 8 - 6x [subtracting 6x from both sides]

⇒ -2x < -8

⇒ 2x > 8 [dividing both sides by - 1 and then inequality sign will change]

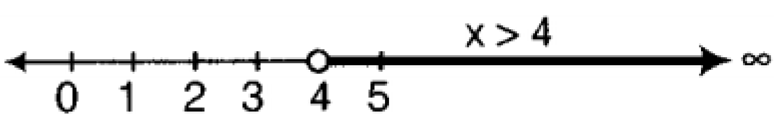
⇒ $\frac{2x}{2} > \frac{8}{2}$ [dividing both sides by 2]

∴ $x > 4$ ...(iii)

Thus, any value of x greater than 4 satisfies the inequality.

∴ Solution set is $x \in (4, \infty)$

The representation of solution of inequality (i) is



From inequality (ii), we get

$\frac{2x-3}{4}+6\geq 2+\frac{4x}{3}\Rightarrow\frac{2x-3+24}{4}\geq\frac{6+4x}{3}$

$\Rightarrow\quad\frac{2x+21}{4}\geq\frac{6+4x}{3}\ \Rightarrow 3(2x+21)\geq 4(6+4x)$

$\Rightarrow\quad 6x+63\geq 24+16x$

$\Rightarrow -10x\geq -39\Rightarrow 10x\leq 39$

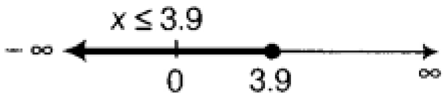
$\Rightarrow\quad\frac{10x}{10}\leq\frac{39}{10}$

$\Rightarrow\quad x\leq 3.9$ ...(iv)

Thus, any value of x less than or equal to 3.9 satisfies the inequality.

$\therefore$ Solution set is $x\in(-\infty,3.9]$.

Its representation on number line is



From Eqs. (iii) and (iv), it is clear, that there is no common value of x, which satisfies both inequalities (iii) and (iv).

Hence, the given system of inequalities has no solution.

35. We know,

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos x = \pm\sqrt{1-\frac{5}{9}} = \pm\frac{2}{3}$

Since, $x\in\left(\frac{\pi}{2},\pi\right)$

cos x will be negative in second quadrant

therefore. $\cos x = -\frac{2}{3}$

We know,

$\cos 2x = 2\cos^2 x - 1$

$\cos\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\cos\frac{x}{2}=\pm\sqrt{\frac{1+\left(\frac{-2}{3}\right)}{2}}=\pm\sqrt{\frac{1}{6}}$

Since, $x\in\left(\frac{\pi}{2},\pi\right)\Rightarrow\frac{x}{2}\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

$\cos\frac{x}{2}$ will be positive in 1st quadrant.

So, $\cos\frac{x}{2}=\frac{1}{\sqrt{6}}$

We know,

$\cos 2x = 2\cos^2 x - 1$

$\cos x = 1 - 2\sin^2\frac{x}{2}$ ... $[\because \cos x = -\frac{2}{3}]$

$-\frac{2}{3} = 1 - 2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2}=\frac{2}{3}+1$

$2\sin^2\frac{x}{2}=\frac{2+3}{3}$

$\sin^2\frac{x}{2}=\frac{5}{6}$

$\sin^2\frac{x}{2}=\pm\sqrt{\frac{5}{6}}$

Since, $x\in\left(\frac{\pi}{2},\pi\right)\Rightarrow\frac{x}{2}\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

$\sin\frac{x}{2}$ will be positive in 1st quadrant

So,

$\sin\frac{x}{2}=\sqrt{\frac{5}{6}}$

We know,

$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$\tan\frac{x}{2}=\frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}}$

$\tan\frac{x}{2} = \sqrt{5}$

Hence, values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$, $\tan\frac{x}{2}$ are $\frac{1}{\sqrt{6}}$, $\sqrt{\frac{5}{6}}$ and $\sqrt{5}$

OR

We have to prove that $\cos 36^o \cos 42^o \cos 60^o \cos 78^o = \frac{1}{16}$.

LHS = $\cos 36^o \cos 42^o \cos 60^o \cos 78^o$

By regrouping the LHS and multiplying and dividing by 2 we get,

$= \frac{1}{2}\cos 36^o \cos 60^o (2\cos 78^o \cos 42^o)$

But 2 cos A cos B = cos (A + B) + cos (A - B)

Then the above equation becomes,

$= \frac{1}{2}\cos 36^o \cos 60^o (\cos(78^o + 42^o) + \cos(48^o - 42^o))$

$= \frac{1}{2}\cos 36^o \cos 60^o (\cos(120^o) + \cos(36^o))$

$= \frac{1}{2}\cos 36^o \cos 60^o (\cos(180^o - 60^o) + \cos(36^o))$

But $\cos(90^o - \theta) = \sin\theta$ and $\cos(180^o - \theta) = -\cos(\theta)$.

Then the above equation becomes,

$= \frac{1}{2}\cos 36^o \cos 60^o (-\cos(60^o) + \cos(36^o))$

Now, $\cos(36^o) = \frac{\sqrt{5}+1}{4}$

$\cos(60^o) = \frac{1}{2}$

Substituting the corresponding values, we get

$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right)$

$= \left(\frac{\sqrt{5}+1}{16}\right)\left(\frac{\sqrt{5}+1-2}{4}\right)$

$= \left(\frac{(\sqrt{5})^2 - 1^2}{16 \times 4}\right)$

$= \left(\frac{5-1}{64}\right)$

$\frac{1}{16}$

LHS = RHS

Hence proved.

Section E

36. i. Number of relations = 2^{mn}

$= 2^{3\times 6} = 2^{18}$

ii. Number of relations = 2^{mn}

$= 2^{2\times 2} = 2^4 = 16$

iii. R = {(x, y): $x \in P$, $y \in Q$ and x is the square of y}

OR

Here, W denotes the set of whole numbers.

We have 2a + b = 5 where $a, b \in W$

$\therefore a = 0 \Rightarrow b = 5$

$\Rightarrow a = 1 \Rightarrow b = 5 - 2 = 3$

and $a = 2 \Rightarrow b = 1$

For a > 3, the values of b given by the above relation are not whole numbers.

$\therefore A = \{(0, 5), (1, 3), (2, 1)\}$

37. i. Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

= 0.05 + 0.10 - 0.02 = 0.13

ii. Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

Probability of atleast one of them does not qualify

$= P(E_1' \cup E_2') = P((E_1 \cap E_2)')$

$= 1 - p(E_1 \cap E_2) = 1 - 0.02 = 0.98$

iii. Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

$= P(E_1' \cap E_2') = P\left((E_1 \cup E_2)'\right)$

= 1 - P($E_1 \cup E_2$) = 1 - 0.13 = 0.87

OR

Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

The probability that Vinod will not qualify the exam.

Probability that only one of them will qualify the exam = P((E_1 - E_2) ∪ (E_2 - E_1))

= P(E_1 - E_2) + P(E_2 - E_1)

= P($E_1 \cup E_2$) - P($E_1 \cap E_2$)

= 0.13 - 0.02 = 0.11

38. i. Let z = 1 + 2i

$\Rightarrow |z| = \sqrt{1+4} = \sqrt{5}$

Now, f(z) = $\frac{7-z}{1-z^2} = \frac{7-1-2i}{1-(1+2i)^2}$

= $\frac{6-2i}{1-1-4i2-4i} = \frac{6-2i}{4-4i}$

= $\frac{(3-i)(2+2i)}{(2-2i)(2+2i)}$

= $\frac{6-2i+6i-2i^2}{4-4i^2} = \frac{6+4i+2}{4+4}$

= $\frac{8+4i}{8} = 1 + \frac{1}{2}i$

f(z) = 1 + $\frac{1}{2}$

∴ |f(z)| = $\sqrt{1+\frac{1}{4}} = \sqrt{\frac{4+1}{4}} = \frac{\sqrt{5}}{2} = \frac{|z|}{2}$

ii. Given that: (z + 3)($\bar{z}$ + 3)

Let z = x + yi

So (z + 3)($\bar{z}$ + 3) = (x + yi + 3)(x - yi + 3)

= [(x + 3) + yi][(x + 3) - yi]

= $(x + 3)^2 - y^2i^2$

= $(x + 3)^2 + y^2$

= $|x + 3 + iy|^2$

= $|z + 3|^2$

iii. The conjugate of -6 - 24i is -6 + 24i.

It is given that -6 + 24i = (x – iy) (3 + 5i)

-6 + 24i = 3x + 5xi - 3iy -5yi^2

-6 + 24i = (3x + 5y) + i(5x - 3y)

Comparing the real and imaginary parts,

3x + 5y = -6

5x - 3y = 24

Solving these two equations we get x = 3 and y = -3.

Therefore, x = 3 and y = -3

Then x + y = 3 - 3 = 0

OR

z = 3 + 4i

$\Rightarrow \bar{z}$ = 3 - 4i